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DIFFRACTION OF A STRONG SHOCK WAVE ON A CYLINDER WITH A TIME-VARYING RADIUS

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Problems of the diffraction of shock waves have been urgent until recently. Whithem [1, 2] suggested an approximate theory for describing strong, non-one-dimensional shocks based on a "rule of characteristic curves" using nonlinear, orthogonal ray coordinates. The author herself [2] acknowledged that "... possibly the most severe test of this theory was its application to diffraction on a round cylinder, which was carried out by Bryson and Gross [3]." Mach reflection (the formation of three shocks and a contact discontinuity) was investigated in [3] and it was shown that a satisfactory description of the Mach stem in terms of the method of [1, 2] is obtained when the influence of the reflected wave and the contact discontinuity wave is neglected.

In the present paper we generalize the problem of [3] to the case of a cylinder whose radius varies fairly slowly with time. Let $r = a(t)$ be the equation for the surface of the cylinder (t is time), the axis of symmetry of which is oriented along the **z** vector. The primary (incident) strong shock is assumed to move at a velocity $\mathbf{u}_0 = u_0\mathbf{e}_x = \text{const}$ ($|\mathbf{e}_x| = 1$, $u_0 >> c$, c being the speed of sound). We solve the problem in a local Cartesian coordinate system (below it is marked by a prime), associated with the midpoint of the Mach stem [the point $N(t)$ in Fig. 1]. In this system the mutual velocity of the shock and the point N is

$$\mathbf{u}_0'(t,\varphi) = \mathbf{u}_0 - \mathbf{v}_0(t,\varphi), \tag{1}$$

where $\mathbf{v}_0 = \frac{\partial}{\partial t}\Big\{(-\mathbf{e}_x \cos\varphi + \mathbf{e}_y \sin\varphi)\Big[a(t) + \frac{1}{2}\, b(t,\varphi)\Big]\Big\}$; φ is the polar angle; $b = 2(NT)$ is the length of the stem (T is the coordinate of the end of the stem in the **x**, **y** coordinate system). In our approximation we assume that the Mach stem is straight and is oriented along a radius. In the primed coordinate system the angle $Q_0'(t, \varphi)$ between the normal to the shock front and the **x** axis is found from the equation

$$\cos Q_0' = (u_0 + v_0(t)\cos\varphi)[(u_0 + v_0\cos\varphi)^2 + v_0^2 \sin^2\varphi]^{-1/2}.$$

The description is tied in to the local coordinate system. We will be interested only in the situation in which the shock wave front has reached the point N. So for t in (1) we take the time that the front "encounters" the point N,

$$t = \Big\{R_0 - \Big[a(t) + \frac{1}{2}\, b(t,\varphi)\,\mathrm{tg}\,\varphi\Big]\Big\} u_0^{-1} \tag{2}$$

[$R_0 = \text{const} \geq (a + b)$ is the distance from the incident front to the origin of coordinates (the point O in Fig. 1) at the time $t = 0$]. Equation (2) serves for determining $t = t(\varphi)$, and one obtains a representation of u_0' as a function of the angle φ:

$$\mathbf{u}_0' = \mathbf{u}_0'(t(\varphi),\varphi) = \mathbf{u}_0'(\varphi).$$

We consider only the case of slow time variation of the cylinder's radius $a(t)$. We confine ourselves to the situation in which $|\mathbf{v}_0| << u_0$, so the function $u_0'(t, \varphi)$ will vary slowly with respect to the first argument.

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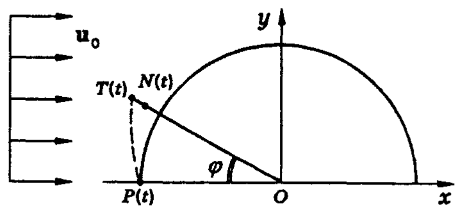

Fig. 1

We introduce an orthogonal nonlinear system of ray coordinates α', β', associated with the point $N(t)$ at the shock wave front ($\alpha \perp \beta$):

$$(\mathrm{M}'d\alpha')^2 + (A'd\beta')^2 = (dx)^2 + (dy)^2. \tag{3}$$

Here $\mathrm{M}' = u'c_0^{-1}$ is the Mach number; u' is the velocity of the shock front; c_0 is the linear speed of sound; A' is the dimensionless cross-sectional area of the ray tube.

The representation $A_1' = ba_0^{-1}$, $a_0 \equiv a(0)$, is valid for the Mach stem, while for the incident shock wave the area A_0' corresponds to the length of the primary shock front between the point $T(t)$ (see Fig. 1) and the x axis:

$$A_0' = a_0^{-1}(a+b)\sin\varphi(\cos Q_0')^{-1}.$$

In irregular waveguides and, in particular, in ray tubes of variable cross section there is a relationship between A' and M' for strong shocks [2]:

$$A_1'(A_0')^{-1} \approx (\mathrm{M}_0')^n(\mathrm{M}_1')^{-n}, \quad n = 1 + 2\gamma^{-1} + [2\gamma(\gamma-1)^{-1}]^{1/2} \tag{4}$$

(γ is the ratio of specific heats).

For the α', β' coordinate system satisfying the condition (3), the following eikonal equation is valid:

$$|\nabla'\alpha'| = (\mathrm{M}')^{-1}. \tag{5}$$

Rays associated with the Mach stem are described by the equations

$$(\mathrm{M}_1')^{-1}\frac{\partial x'}{\partial \alpha'} = \cos Q_1', \quad (\mathrm{M}_1')^{-1}\frac{\partial y'}{\partial \alpha'} = \sin Q_1',$$

where Q_1' is the angle between the vectors $\mathbf{u}_1'$ and $\mathbf{x}$.

In the transition from the primary (incident) wave to the secondary wave (its front coincides with the Mach stem), a bend occurs in the shock front. This corresponds to the fact that the ray vector α' changes abruptly to conserve the absolute value $|\alpha'| = \mathrm{const}$:

$$\alpha_1' = \alpha_0' \approx [a - (a+b)\cos\varphi](\mathrm{M}_0'\cos Q_0')^{-1},$$
$$\mathrm{M}_0' = \mathrm{M}_0[(1 + v_0u_0^{-1}\cos\varphi)^2 + (v_0u_0^{-1}\sin\varphi)^2]^{1/2}.$$

Equation (5) is used in the approximation

$$(R_1')^{-1}\frac{\partial \alpha'}{\partial \varphi} \approx (\mathrm{M}')^{-1} \tag{6}$$

(R_1' is the radius vector of points on the Mach stem). In the vicinity of the point $N(t)$ we have $R_1' = a + \tfrac{1}{2}b$. Equations (4) and (6) consist of a closed system that reduces to a differential equation for the size $b[t(\varphi), \varphi]$ of the Mach stem:

$$b = (a+b)(\sin\varphi)^{n+1}[1 + (v_0\sin\varphi)^2(u_0 + v_0\cos\varphi)^{-2}]^{1/2}\,\Pi(b, a, \varphi). \tag{7}$$

Here

$$\Pi \equiv \left\{ M_0' \left[(a + \frac{1}{2} b) \sin\varphi \right]^{-1} \frac{\partial}{\partial\varphi} [(a - (a+b)\cos\varphi)(M_0' \cos Q_0')^{-1}] \right\}^n .$$

The boundary condition for the function b in Eq. (7) has the form $b[t(0), 0] = 0$. The function Π varies slowly compared with the factor $(\sin\varphi)^{n+1}$ in (7). For $a = a_0 = \text{const}$ and $\varphi \to 0$ we have $\Pi \approx 1$. The function $b = b_0(\varphi)$ has been calculated in [3] for $a = a_0$ and $\varphi \leq 60°$, and there is also a graph of the dimensionless function $b_0(\varphi)(a_0)^{-1}$ in [2] (Fig. 8.13). The condition of smallness of the Mach stem, $b \ll a$, is satisfied for slow time variation of the cylinder's radius ($|v_0| \ll u_0$) and for low amplitudes of that variation ($|a - a_0|a_0^{-1} \ll 1$). We can therefore solve Eq. (2) by the method of successive approximations, taking for the first approximation the equation

$$\begin{aligned} b \approx B_1(\varphi) = (a + b_0(\varphi))(\sin\varphi)^{n+1} \times \\ \times [1 + (v_0 \sin\varphi)^2 (u_0 + v_0 \cos\varphi)^{-2}]^{1/2} \Pi(b_0(\varphi), a, \varphi). \end{aligned} \tag{8}$$

The right side of Eq. (8) is a known function, since $b_0(\varphi)$ has been found in [3], the forms of $a(t)$ and $v_0 = da/dt$ are given by the conditions of the problem, and the relationship between t and φ is given by Eq. (2). The second approximation for b is obtained by substituting $b = B_1$ into the right side of (7).

Using the method of successive approximations to solve (7) thus reduces the approximate determination of $b[t(\varphi), \varphi]$ to a problem of differentiation with respect to φ of functions that are known at each step.

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